Optimal Labeling Searching based on Classification of Unique Mapping for BICM-ID with 8-APSK

Ben Zhang, Kewu Peng, Jian Song, and Junhao Lin
Tsinghua National Laboratory for Information Science and Technology (TNList)
Department of Electronic Engineering, Tsinghua University, Beijing 100084, China
Email: nebgnahz@gmail.com, {pengkewu, json}@tsinghua.edu.cn, jasonlin54321@gmail.com

***Abstract*—Powerline Communication is promising but problems caused by bad channel responses arise the question of robust coded modulation scheme. The model of BICM-ID system can be adopted to PLC network and used for performance analysis. To achieve optimal labeling search, a new and simplified criterion – Hamming-Euclidean Distance Spectra – is proposed to obtain the classification of unique mappings. 8-PSK in previous works will be investigated as an example to illustrate the effectiveness of this criterion, and the number of unique mappings can be cut down considerably from 40320(=8!) to only 70. This paper mainly focuses on the BICM-ID system with 8-APSK, where the input is more Gaussian-like than that of 8-PSK, therefore non-negligible shaping gain can be expected. For 8-APSK, the number of unique mappings is cut down to only 92. With this small set of unique mappings, it is possible to traverse all the labelings and optimal labeling can be obtained. As an example, the design of BICM-ID systems with 8-APSK is accomplished with the aid of EXIT charts. The results outperform the previous schemes and we hope this improvements would be considered in the standardization of PLC.**

***Index Terms*—Classification of Unique Mappings, 8-APSK mapping, Hamming-Euclidean Distance Spectra, BICM-ID, EXIT charts**

I. INTRODUCTION

Due to its deep penetration and utilization of previous wires, Powerline Communication (PLC) is now drawing researchers interests to its standardization and performance analysis. However, it is also well known that the PLC channel varies vastly with frequency, location, time and the type of equipment connected to the channel. To overcome such problems and build a reliable and robust system over PLC, many coded modulation schemes, such as LDPC and convolution code to encode bits and ROBO [1], QPSK(DQPSK), 8PSK(D8PSK) to obtain corresponding symbols, are proposed these years. Bit-interleaved coded modulation (BICM), suggested in [2], can be considered as a serially concatenated code, where the constellation mapper performs as the inner code. BICM with iterative decoding (BICM-ID), based on exchanging extrinsic information between the demapper and the decoder, was first proposed in [3]–[5]. This communication model can also be applied to PLC to achieve better performance. Previous analysis over BICM-ID system pointed out that the choice of constellation and labeling are crucial in BICM-ID system [6], [7], meanwhile the optimal labeling is typically very difficult to find due to the big size of labeling space. Unlike in BICM, conventional Gray mapping typically will not guarantee to perform the best in BICM-ID systems. Such conclusions can be easily observed with the powerful tool– Extrinsic Information Transfer (EXIT) charts, which was first proposed by ten Brink in [8]. In the previous labeling searching methods, some are based on EXIT-chart analysis [9], while other techniques are based on the Euclidean distance properties [10], [11]. However, these searching methods are not generic, and they cannot provide a systematic or complete searching in the labeling space. It is Schreckenbach *et al* who came up with a systematic searching method known as binary switch algorithm (BSA) [6]. For high-order constellations, however, only near optimal mapping could be searched for BICM-ID even with BSA. While for low-order constellations, e. g. 8-PSK, it is possible to enumerate and traverse all the possible mappings because some mappings are equivalent and computational costs can be cut down significantly. The equivalence of mappings and labelings has been discussed in [12] with four operators named constellation rotation (Cr), constellation reflection (Cf), sub-mapping reordering (Sr), and bit flipping (Bf) are proposed. The paper then confirmed that the number of unique mappings is only 86 and conjectured it should be 70 for 8-PSK.

In this paper, our objective is to propose a new method of labeling classification, which is based on the conjecture for the equivalence of labelings in BICM or BICM-ID systems. This new criterion of Hamming-Euclidean Distance Spectra is more essential than those mapping operators suggested in [12]. And from this perspective of view, 70 unique mappings can be easily derived for 8-PSK. However in this paper, we are more concerned about 8-APSK since 8-APSK has more Gaussian-like input and will provide considerably shaping gain. By analyzing the Hamming-Euclidean Distance Spectra of 8-APSK mapping, we obtain only 92 unique mappings. Then with the help of EXIT charts, we are able to find an optimal labeling for the BICM-ID system by traversing this small set of unique mappings instead of using BSA algorithm or searching in the large labeling space. Here the criterion of an optimal labeling is according to the analytical result of EXIT charts, i. e. whether there exists an EXIT tunnel between the demapper's curve and inverted decoder's curve, which indicates it is possible to decode without error at the lowest possible SNR. As an example of our new labeling searching method, we design a coded modulation scheme for BICM-ID with 8-APSK and provide corresponding simulation results.

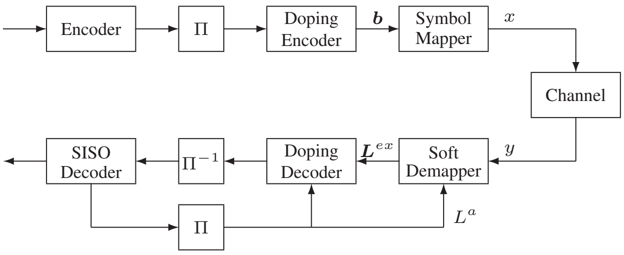

Fig. 1. BICM-ID system model with doping

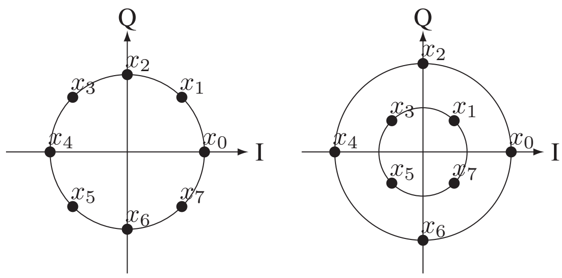

Fig. 2. Examples of constellations, 8-PSK and 8-APSK

This paper is organized as follows. In Section II, the system model is introduced, including the brief review of BICM-ID system and labeling function. The analysis of labeling equivalence and the conjecture are discussed in Section III, together with the example of 8-PSK mapping and 8-APSK mapping. Section IV mainly concentrates on 8-APSK labeling design for an example of BICM-ID system based on classification of unique mappings and aided with the EXIT charts. Simulation results of bit error rate (BER) and corresponding EXIT analytical results are provided in Section V, which is followed by concluding remarks in Section VI.

II. System Model

The baseband system model for BICM-ID system is shown in Fig. 1, where the doping technique is included in order to avoid the high error-floor in traditional BICM-ID systems.

At the transmitter side, the information bits are encoded, bit-interleaved by Π, and doping coded with doping rate *P*, i. e. every *P*th bit is coded into a unit-rate two-state recursive systematic convolutional (RSC) code. Then every *m* bits $\boldsymbol{b} = [b_{m-1}, b_{m-2}, \ldots, b_0]$ are mapped to a symbol $x = \mu(\boldsymbol{b})$ by the symbol mapper. We will use $\mathbb{X}$ to represent the set of 2^m symbols on the constellation mapping space and $x \in \mathbb{X}$.

The symbols are then transmitted over a baseband equivalent channel, which could be additive white Gaussian noise (AWGN) or fading channel. At the receiver side, unlike BICM system, the extrinsic soft values are fed back from the soft-input-soft-output (SISO) decoder to the soft demapper as *a priori* information. Usually the extrinsic soft values are in the form of the log-likelihood ratios (LLRs) $\boldsymbol{L^a} = [L^a_{m-1}, L^a_{m-2}, \ldots, L^a_0]$. The SISO demapper calculates LLR of each bit to facilitate iterative decoding, and the formula for the LLR of *n*th bit is:

$$L^{ex}_n = \log \frac{\sum_{x \in \chi^{(0)}_n} p(y|x) Pr(x|L^a))}{\sum_{x \in \chi^{(1)}_n} p(y|x) Pr(x|L^a))} - L^a_n \qquad (1)$$

where $\chi^{(b)}_n = \{x \in \mathbb{X}, \boldsymbol{b} = [b_{m-1}, b_{m-2}, \ldots, b_0]\} = \mu^{-1}(x)|b_n = b\}, b \in \{0, 1\}$ is the subset of $\mathbb{X}$, $p(y|x)$ denotes the conditional probability density of receiving symbol y when the symbol x is transmitted, and the $Pr(x|L^a)$ is the conditional probability of symbol x when *a priori* LLRs of those *m* bits are given. During the transfer process of extrinsic information between the demapper and the decoder, it is necessary to include the doping decoder, Π and Π^{-1}.

In the mapping procedure, the symbol $x \in \mathbb{X}$ might be a real number, a complex number, or even a 3- or 4-dimensional symbol. Two main constellation maps (8-PSK and 8-APSK) are depicted in Fig. 2 as examples and will be discussed later. In this paper, 8-APSK constellation is composed of two concentric rings, each with uniformly 4 spaced PSK points. The constellation map has been normalized with the radius ratio of the outer ring and inner ring being 1.90.

As has been defined, the labeling function μ maps $\boldsymbol{b}$ of *m* bits onto a symbol $x \in \mathbb{X}$. There are totally $\mathbf{M} = \|\mathbb{X}\| = 2^m$ symbols $\{x_0, x_1, \ldots, x_i, \ldots, x_{M-1}\}$, and labeling can be denoted by a vector $\boldsymbol{\alpha} = \{\alpha_0, \alpha_1, \ldots, \alpha_i, \ldots, \alpha_{M-1}\}$, which consists of the permutation of integer number 0 to $M-1$. The element α_i in this vector links the $\boldsymbol{b}$ ($\boldsymbol{b}$ is the binary representation of a decimal number α_i) with the symbol x_i. From this perspective, there seems to be $M!$ possibilities of labeling at the first glance and the labeling space can be viewed as the permutation of $\boldsymbol{\alpha} = \{0, 1, \ldots, M-1\}$; but after exploiting the properties of labelings discussed in the next section we can cut down the possibilities considerably, which will greatly facilitate the optimal design of BICM-ID scheme by using the exhaustive search.

III. Properties of labeling

A. Equivalence Analysis

When considering the equivalence of different labelings in BICM-ID systems, we figure out that equivalent labelings will lead to identical bit error performance. At the receiver side, the pair-wise error event probability $Pr(x_i \to x_j)$ is determined by the Euclidean distance D_{ij} between symbols x_i and x_j. But since the transferred soft information in the receiver side is in bits for BICM-ID systems, we are more concerned about the pair-wise bit-error probabilities, i. e., $H_{ij} \times Pr(x_i \to x_j)$, where the H_{ij} is the Hamming distance between $\mu^{-1}(x_i)$ and $\mu^{-1}(x_j)$. From this perspective, for certain labeling of $\boldsymbol{\alpha}$, the Hamming distance and Euclidean distance will jointly determine the bit error performance. For the Euclidean distance D_{ij} of a constellation mapping, there are totally n different distances denoted by $d_1, d_2, \ldots, d_n$. To further discuss the insight of the labeling, we define the Hamming-Euclidean Distance Spectra below and denote it by matrix S.

$$S = [S_{uv}]_{m\times n} = \begin{bmatrix} S_{11} & \dots & S_{1n} \\ \vdots & \ddots & \vdots \\ S_{m1} & \dots & S_{mn} \end{bmatrix} \quad (2)$$

The element S_{uv} is the number of pairs (x_i, x_j) whose Euclidean distance is d_v and corresponding Hamming distance between $\mu^{-1}(x_i)$ and $\mu^{-1}(x_j)$ is u for all possible (i, j); thus the whole matrix represents the Hamming Euclidean Distance Spectra corresponding to a labeling. For example, in 8-PSK with natural labeling $\alpha = \{0, 1, 2, 3, 4, 5, 6, 7\}$, there are three different Hamming distances and four different Euclidean distances. Therefore the 3×4 matrix is

$$S = \begin{bmatrix} 4 & 4 & 0 & 4 \\ 2 & 4 & 6 & 0 \\ 2 & 0 & 2 & 0 \end{bmatrix}$$

and the element $S_{11} = 4$ means that there are four pairs $(x_0, x_1), (x_2, x_3), (x_4, x_5), (x_6, x_7)$ whose Hamming distances are all 1 and Euclidean distances are d_1. Other elements can be similarly calculated by counting corresponding pairs.

Then based on the above analysis, we reach the following conjecture: labelings that have the same Hamming-Euclidean Distance Spectra are equivalent in BICM or BICM-ID system.

B. Approach to 8-PSK unique mappings

As being introduced in Section I, the four operators (Cr, Cf, Sr, Bf) are useful in classifying equivalent labelings. While in our opinion, the validation of these four operators can be easily derived from the proposed Conjecture.

For constellation rotation or reflection, the relative positions of the constellation points keep unchanged, thus it is easy to see their Hamming Euclidean Distance Spectra remains the same. While for sub-mapping reordering and bit flipping, it is also obvious that the Hamming Euclidean Distance Spectra does not change. Actually, the properties behind these four operators are sufficient conditions for the equivalence of two labelings according to the conjecture. Therefore we only need to consider the 86 candidates resulted from their works in [12] instead of the whole labeling space. We calculate Hamming-Euclidean Distance Spectra for each of them and classify those with the same matrix S into a group; finally we reach only 70 different matrixes S, i. e. 70 unique mappings for 8-PSK.

The reason we can further reduce the number of unique mappings is due to the fact that there are labeling pairs that have the same Hamming-Euclidean Distance Spectra but neither can be derived from the other via those four operators. Fig. 3 shows one example of such pairs, and their equivalence will be further verified in Section V with EXIT analysis.

C. Applying to 8-APSK mapping

When it comes to 8-APSK mapping, those four operators (Cr, Cf, Sr, Bf) can also be applied with minor modifications on Cr operator. As the rotational symmetry of 8-APSK is different from 8-PSK–the rotation of $\frac{n\pi}{2}$ gives the same

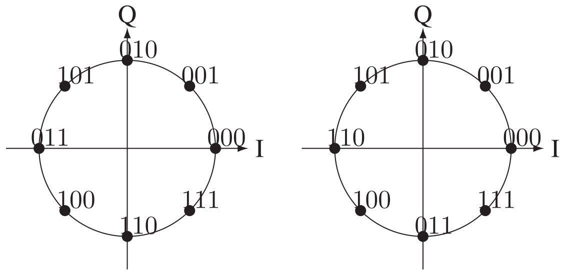

Fig. 3. Example of one pair of labelings in 8-PSK. $\boldsymbol{\alpha}_1 = \{0, 1, 2, 5, 3, 4, 6, 7\}$ and $\boldsymbol{\alpha}_2 = \{0, 1, 2, 5, 6, 4, 3, 7\}$ having the same Hamming-Euclidean Distance Spectra $S = \begin{bmatrix} 3 & 4 & 3 & 2 \\ 2 & 4 & 4 & 2 \\ 3 & 0 & 1 & 0 \end{bmatrix}$

constellation, rather than $\frac{n\pi}{4}$ in the case of 8-PSK. So the Cr operator will only give a set expansion of four rather than eight. This method then classifies the 40320(=8!) possibilities into 120 unique mappings. And again after the exhaustive search based on our conjecture, it turns out that only 92 of these 120 will have unique Hamming Euclidean Distance Spectra, indicating 92 possible different kinds of bit error performance for a given system. These labelings are listed in Table I for reference and further use.

IV. Design Of BICM-ID Based On Classification Of Unique Mappings And EXIT Charts

In previous sections, we have obtained the unique mappings for a given low-order constellation using the criterion based on Hamming-Euclidean Distance Spectra. Now we will show how to apply the property of unique mappings to the design of BICM-ID system shown in Fig. 1.

To design a BICM-ID system with good performance, the EXIT curve-fitting method helps choose the well-matched labeling for the outer channel code. From this point, the ultimate goal is to search for a labeling to let the demapper's EXIT curve fit the inverted decoder's EXIT curve. That is, there exists an EXIT tunnel while requiring lowest possible SNR. Since we are interested in the BICM-ID systems with doping technique, we will use doped demapper's EXIT curve to analyze the system behavior. For each labeling, there exists a minimum SNR, or SNR threshold. If the actual SNR is smaller than this threshold, then the doped demapper's EXIT curve will not be able to be above the inverted decoder's curve. For all the labelings, we will find the minimum value of SNR threshold, and its corresponding labeling is expected to be the optimal labeling for the target BICM-ID system.

Such searching in the labeling spaces seems brutal and time-consuming, but based on our previous discussion on the labelings equivalence and the classification of unique mappings, the labeling search could be simplified greatly, e. g. , only 70 unique mappings need to be investigated for 8-PSK, and 92 for 8-APSK.

We then consider the 92 unique 8-APSK mappings for a specific BICM-ID system, where the outer code is a rate-

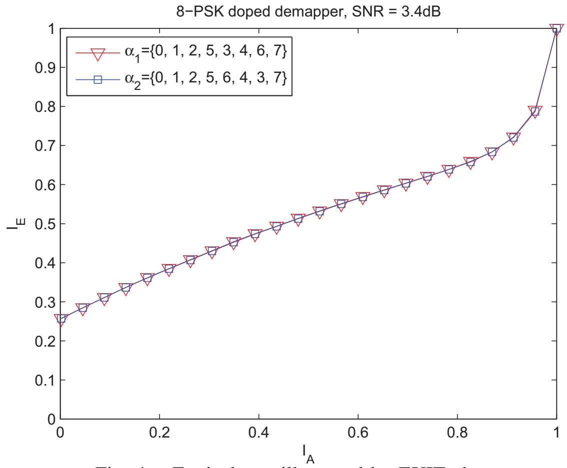

Fig. 4. Equivalence illustrated by EXIT charts.

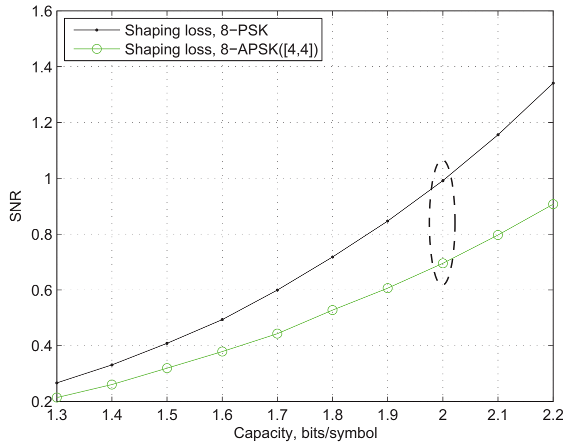

Fig. 6. Shaping loss of 8-PSK and 8-APSK compared to Shannon limit with Gaussian input

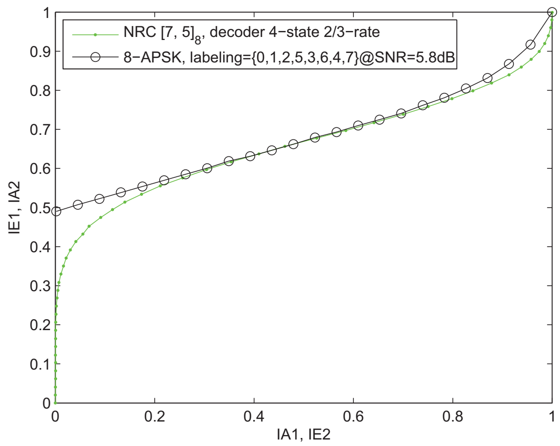

Fig. 5. Performance of designed system, illustrated by EXIT charts analysis

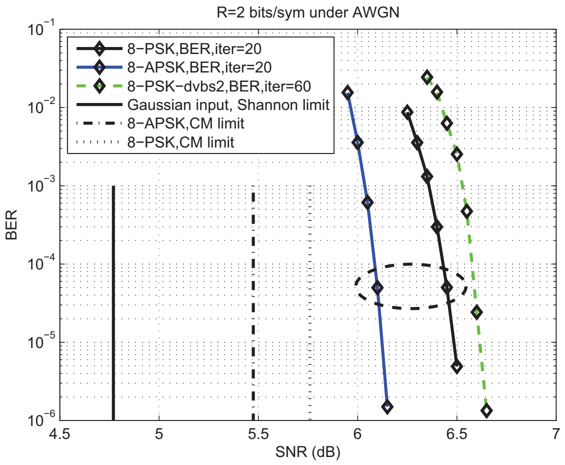

Fig. 7. Performance of designed system, illustrated by BER vs. SNR curve.

2/3 4-state non-recursive convolutional (NRC) code with the generators of $[7, 5]_8$ in octal. Besides, the doping rate for this system is set as $P = 50$. Under this condition, we propose that the labeling $\boldsymbol{\alpha} = \{0, 1, 2, 5, 3, 6, 4, 7\}$ is the optimal labeling for this BICM-ID system with 8-APSK after the exhaustive labeling search. Detailed simulation results are presented in the next section.

V. Simulation Results

A. *Verifying the equivalence with EXIT charts*

Fig. 4 depicts the EXIT charts of the two labelings of 8-PSK. And as has been discussed from the constellation map, we cannot directly obtain their equivalence via those four mapping operators. While from the criterion of Hamming Euclidean Distance Spectra, they are expected to have the identical bit error performance. And this EXIT-chart analytical result does validate our conjecture of labeling equivalence.

B. *Performance Evaluation in BICM-ID system*

In Section IV, we suggest that the labeling $\boldsymbol{\alpha} = \{0, 1, 2, 5, 3, 6, 4, 7\}$ be the optimal labeling for the BICM-ID system with 8-APSK. To verify the performance, the EXIT curve of the doped demapper is shown in Fig. 5. Meanwhile the inverted decoder's EXIT curve is also provided to show the fitting of the decoder and the doped demapper. A narrow but clear EXIT tunnel can be observed. In our simulation, a codeword length of 61440 is used and the maximum number of simulated frames is set to be 5000. The bit-interleaver employed is S-random interleaver with the length of 61440 bits, and the parameter S is chosen as 160. Maximum iterative number is set to be 20 and the log-MAP algorithm is employed during the iterative process.

The shaping loss of 8-PSK and 8-APSK compared to Shannon limit with Gaussian input is depicted in Fig. 6. The shaping gain of 8-APSK over 8-PSK at capacity=2 bits/symbol is approximate 0.28 dB. In Fig. 7 where the simulation results of BER performance are shown, our designed system achieves this expected shaping gain. It provides more than 0.3 dB gain than the optimal 8-PSK labeling searched in the similar way. Besides, our system is seen better than the DVB S2 system with 8-PSK.

TABLE I
CLASSIFICATIONS OF UNIQUE MAPINGS OF 8-APSK, 92 OUT OF 40320(=8!) LABELINGS, BASED ON HAMMING-EUCLIDEAN DISTANCE SPECTRA

row	labeling	row	labeling	row	labeling
1	0,1,6,3,4,5,2,7	32	0,1,2,4,6,7,3,5	63	0,1,2,3,6,5,7,4
2	0,1,6,3,4,7,2,5	33	0,1,6,3,5,4,2,7	64	0,1,2,4,5,3,7,6
3	0,1,2,5,4,3,6,7	34	0,1,6,3,5,2,7,4	65	0,1,2,3,5,4,7,6
4	0,3,4,7,1,2,5,6	35	0,1,2,5,4,3,7,6	66	0,1,2,3,5,6,7,4
5	0,1,2,3,4,5,6,7	36	0,1,3,4,2,6,5,7	67	0,3,4,1,7,2,5,6
6	0,1,2,7,4,3,6,5	37	0,1,2,4,5,3,6,7	68	0,1,2,5,7,4,3,6
7	0,1,2,3,4,7,6,5	38	0,1,2,4,5,6,3,7	69	0,1,2,6,4,3,5,7
8	0,1,2,7,4,5,6,3	39	0,1,2,3,4,5,7,6	70	0,1,2,3,4,6,5,7
9	0,1,6,3,4,2,7,5	40	0,1,2,3,4,6,7,5	71	0,1,2,7,4,3,5,6
10	0,1,3,4,2,5,7,6	41	0,1,2,4,5,7,3,6	72	0,1,2,3,4,7,5,6
11	0,1,2,5,4,6,3,7	42	0,1,2,4,5,7,6,3	73	0,1,2,5,7,3,6,4
12	0,1,7,2,4,3,5,6	43	0,1,2,6,4,5,7,3	74	0,1,2,4,7,3,6,5
13	0,1,7,2,4,3,6,5	44	0,1,2,3,5,7,6,4	75	0,1,2,4,7,5,6,3
14	0,1,2,5,4,7,3,6	45	0,3,4,1,6,5,2,7	76	0,1,2,3,7,4,6,5
15	0,1,2,6,4,5,3,7	46	0,1,2,5,6,3,4,7	77	0,1,2,3,7,5,6,4
16	0,1,3,4,2,6,7,5	47	0,1,2,5,6,7,4,3	78	0,1,3,4,7,2,5,6
17	0,1,2,7,4,5,3,6	48	0,1,2,3,6,5,4,7	79	0,1,2,4,3,6,5,7
18	0,1,2,7,4,6,3,5	49	0,1,2,3,6,7,4,5	80	0,1,2,5,7,3,4,6
19	0,1,2,6,4,7,3,5	50	0,1,6,2,5,4,3,7	81	0,1,2,5,7,6,4,3
20	0,1,7,2,4,5,3,6	51	0,1,3,4,5,2,6,7	82	0,1,2,4,3,7,5,6
21	0,1,7,2,4,6,3,5	52	0,1,3,2,5,4,6,7	83	0,1,2,3,6,4,5,7
22	0,1,3,5,4,2,7,6	53	0,1,3,2,5,7,6,4	84	0,1,2,7,6,3,5,4
23	0,1,3,2,4,5,7,6	54	0,1,3,2,6,7,5,4	85	0,1,2,4,6,7,5,3
24	0,1,3,2,4,6,7,5	55	0,1,2,6,5,3,4,7	86	0,1,2,3,7,5,4,6
25	0,1,6,2,5,3,4,7	56	0,1,2,5,3,4,7,6	87	0,1,2,3,6,7,5,4
26	0,1,6,3,5,2,4,7	57	0,1,2,5,3,6,4,7	88	0,1,2,4,7,3,5,6
27	0,1,2,5,3,4,6,7	58	0,1,2,4,3,5,7,6	89	0,1,2,4,7,6,5,3
28	0,1,2,4,3,5,6,7	59	0,1,2,5,3,7,4,6	90	0,1,2,3,7,4,5,6
29	0,1,2,5,6,7,3,4	60	0,1,2,3,5,6,4,7	91	0,1,2,6,7,3,5,4
30	0,1,2,4,3,7,6,5	61	0,1,2,4,6,3,7,5	92	0,1,2,3,7,6,5,4
31	0,1,2,5,3,7,6,4	62	0,1,2,3,5,7,4,6		

VI. CONCLUDING REMARKS

In this paper, we propose Hamming-Euclidean Distance spectra functioning as a criterion to evaluate the equivalence of labelings in BICM or BICM-ID systems and conjecture that labelings with the same Hamming-Euclidean Distance Spectra are equivalent in BICM or BICM-ID system. Based on the proposed criterion and conjecture we can easily obtain unique mappings for the low-order constellation systems. On the other hand, this new conjecture can also help explain some previous investigations of the labeling equivalence perfectly. Simulation results including EXIT charts analysis and BER performance evaluation for BICM-ID systems with 8-APSK demonstrate the effectiveness of this method, i. e., searching for optimal labelings based on the classification of unique mappings is feasible. And such results will effectively aid the design of PLC network and help improve the reliability.

VII. ACKNOLEDGEMENT

This work was supported in part by SGCC - R&D of A High Performance, Low Cost Broad-band PLC Modem IC, and Agilent Grant, No. 2348-CHN11. And the authors would like to thank the helpful comments from anonymous reviewers.

REFERENCES

[1] M. K. Lee, R. E. Newman, H. A. Latchman, S. Katar, and L. Yonge, “Homeplug 1.0 powerline communication lans - protocol description and performance results.”

[2] G. Taricco, G. Caire, and E. Biglieri, “Bit-interleaved coded modulation,” *IEEE Transactions on Information Theory*, vol. 44, no. 3, pp. 927–946, May 1998.

[3] X. Li and J. A. Ritcey, “Bit-interleaved coded modulation with iterative decoding,” *IEEE Commun. Lett.*, vol. 1, no. 6, pp. 169–171, Nov. 1997.

[4] J. Speidel, S. ten Brink, and R. Yan, “Iterative demapping and decoding for multilevel modulation,” *Proc. IEEE Globcom Conference*, pp. 579–584, Nov. 1998.

[5] X. Li and J. A. Ritcey, “Trellis-coded modulation with bit interleaving and iterative decoding,” *IEEE J. Sel. Areas Commun.*, vol. 17, no. 4, pp. 715–724, Apr. 1999.

[6] F. Schreckenbach, J. H. N. Görtz, and G. Bauch, “Optimization of symbol mappings for bit-interleaved coded modulation with iterative decoding,” *IEEE Commun. Letters*, vol. 7, no. 12, pp. 593–595, Dec. 2003.

[7] W. R. Carson, I. Chatzigerorgiou, M. R. D. Rodrigues, I. J. Wassell, and R. Carrasco, “On the performance of iterative demapping and decoding techniques over quasi-static fading channels,” *Proc. PIMRC 2007*, Sep. 2007.

[8] C. Stierstorfer and R. F. H. Fischer, “(gray) mappings for bit-interleaved coded modulation,” *Proc. IEEE Veh. Technol. Conf. (VTC07)*, pp. 1703–1707, Apr. 2007.

[9] T. W. Yu, C. Y. Wang, C. H. Wang, , and W. H. Shen, “Exit-chart based labeling design for bit-interleaved coded modulation with iterative decoding,” *Proc. IEEE Int. Symp. Inf. Theory*, pp. 56–60, Jun. 2007.

[10] J. Tan and G. L. Stüber, “Analysis and design of symbol mappers for iteratively decoded BICM,” *IEEE Trans. Wireless Commun.*, vol. 4, no. 2, pp. 662–672, Mar. 2005.

[11] N. H. Tran and H. H. Nguyen, “Signal mappings of 8-ary constellations for bit interleaved coded modulation with iterative decoding,” *IEEE Trans. Broadcasting*, vol. 52, no. 1, pp. 662–672, Mar. 2005.

[12] W. R. Carson, I. J. Wassell, and M. R. D. Rodrigues, “Optimal 8PSK mappings for BICM-ID over quasi-static fading channels,” 2008.